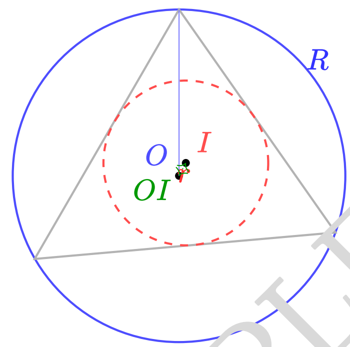
 Name: ______________________________

(ID:1013) 1. The number 2835 can be written as the sum of consecutive positive integers in several ways. When written as the sum of the greatest possible number of consecutive positive integers, what is the largest of these integers?

(ID:1078) 2. A triangle has circumradius $R = 18$ and inradius $r = 8$. Find the distance OI between the circumcenter and incenter.



(ID:1336) 3. Shelby took 4 tests, each worth a maximum of 100 points. The score on each test was an integer between 0 and 100, inclusive. Shelby received the same score on the first 3 tests, and a higher score on the last test. Shelby's average score across all 4 tests was 73. How many different values are possible for Shelby's score on the last test?

(ID:409) 4. What is the units digit of 44^{150}?

(ID:774) 5. What is the remainder when $\underbrace{111\cdots111}_{\text{11 ones}}$ is divided by 30?

(ID:1090) 6. How many ordered sextuples (a, b, c, d, e, f) with each entry in $\{-1, 0, 1\}$ satisfy $a^3 + b^6 + c^4 + d + e^5 + f^2 = 1$?

(ID:334) 7. Find the number of factors of 65^2.

(ID:1298) 8. How many trailing zeros does 105! have?

(ID:1090) 9. How many ordered sextuples (a, b, c, d, e, f) with each entry in $\{-1, 0, 1\}$ satisfy $a^3 + b^5 + c^2 + d^6 + e^4 + f = 3$?

(ID:1087) 10. What is the sum of all values of $a + b$ for unordered pairs of positive integers (a, b) satisfying

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{10}$$

Name: ____________________

Answer Key

1. 75
2. 6
3. 9
4. 6
5. 11
6. 144
7. 9
8. 25
9. 147
10. 327

Solution to Problem 1

Step 1: Understanding Consecutive Sums
For a sequence of k consecutive integers starting at a:

$$a + (a+1) + \cdots + (a+k-1) = ka + \frac{k(k-1)}{2}$$

Multiply both sides by 2 and rearrange:

$$k(2a+k-1) = 2(2835) = 5670$$

This means k and $(2a+k-1)$ are factor pairs of 5670.

Step 2: Identify the Parity Constraint
Notice that $2a$ is always even, so:

- If k is odd, then $2a+k$ is odd, making $(2a+k-1)$ even.
- If k is even, then $2a+k$ is even, making $(2a+k-1)$ odd.

Therefore, k and $(2a+k-1)$ must have **opposite parity**.
Since we need $a \geq 1$, we also require $(2a+k-1) > k$, which means $a \geq 1$.

Step 3: Find Factor Pairs with Opposite Parity
We factor 5670 and look for pairs (k, m) where:
1. $k \cdot m = 5670$
2. k and m have opposite parity (one odd, one even)
3. $k \geq 2$ (at least two consecutive integers)
4. $m > k$ (ensures $a \geq 1$)

For each valid pair, we compute:

$$a = \frac{m-k+1}{2}$$

Step 4: Valid Sequences
$k=2$, $m=2835$ (even × odd): $a=1417$, so $1417+1418=2835$
$k=3$, $m=1890$ (odd × even): $a=944$, so $944+945+946=2835$
$k=5$, $m=1134$ (odd × even): $a=565$, so $565+566+\cdots+569=2835$
$k=6$, $m=945$ (even × odd): $a=470$, so $470+471+\cdots+475=2835$
$k=7$, $m=810$ (odd × even): $a=402$, so $402+403+\cdots+408=2835$
$k=9$, $m=630$ (odd × even): $a=311$, so $311+312+\cdots+319=2835$
$k=10$, $m=567$ (even × odd): $a=279$, so $279+280+\cdots+288=2835$
$k=14$, $m=405$ (even × odd): $a=196$, so $196+197+\cdots+209=2835$
$k=15$, $m=378$ (odd × even): $a=182$, so $182+183+\cdots+196=2835$
$k=18$, $m=315$ (even × odd): $a=149$, so $149+150+\cdots+166=2835$
$k=21$, $m=270$ (odd × even): $a=125$, so $125+126+\cdots+145=2835$
$k=27$, $m=210$ (odd × even): $a=92$, so $92+93+\cdots+118=2835$
$k=30$, $m=189$ (even × odd): $a=80$, so $80+81+\cdots+109=2835$
$k=35$, $m=162$ (odd × even): $a=64$, so $64+65+\cdots+98=2835$
$k=42$, $m=135$ (even × odd): $a=47$, so $47+48+\cdots+88=2835$

$k = 45$, $m = 126$ (odd $\times$ even): $a = 41$, so $41 + 42 + \cdots + 85 = 2835$
$k = 54$, $m = 105$ (even $\times$ odd): $a = 26$, so $26 + 27 + \cdots + 79 = 2835$
$k = 63$, $m = 90$ (odd $\times$ even): $a = 14$, so $14 + 15 + \cdots + 76 = 2835$
$k = 70$, $m = 81$ (even $\times$ odd): $a = 6$, so $6 + 7 + \cdots + 75 = 2835$

Step 5: Find Maximum Length
The greatest number of consecutive integers occurs with the largest valid k.
The maximum is $k = 70$ starting at $a = 6$:

$$6 + 7 + \cdots + 75 = 2835$$

This sequence has 70 consecutive integers.

Step 6: Find the Largest Integer
The largest integer in this sequence is:

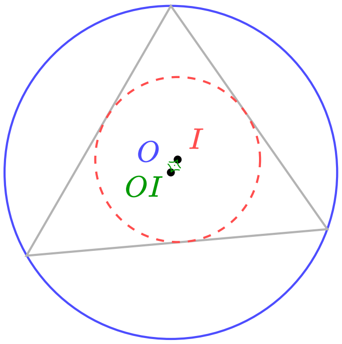
$$a + k - 1 = 6 + 70 - 1 = 75$$

Step 7: Final Answer
The largest of these integers is:

$$\boxed{75}$$

Name: ______________________________

Solution to Problem 2



Step 1: State Euler's Formula
Euler's formula: $OI^2 = R(R - 2r)$, where R is the circumradius, r the inradius, and OI the distance between the circumcenter and incenter.

Step 2: Substitute and Solve

$$OI^2 = 18(18 - 2 \cdot 8) = 18(2) = 36$$
$$OI = \sqrt{36} = 6$$

Step 3: Final Answer

$$\boxed{6}$$

Solution to Problem 3

Step 1: Express the Last Score in Terms of s
Let $s =$ Shelby's score on each of the first 3 tests and $L =$ the last test score.
The average across all 4 tests is 73, so the total score is $4 \times 73 = 292$:

$$3s + L = 292$$
$$L = 292 - 3s$$

Step 2: Find the Valid Range for s
Two constraints must both hold: $L > s$ (last score is strictly higher) and $0 \leq L \leq 100$.

$$L > s \implies 292 - 3s > s \implies 292 > 4s \implies s < 73$$
$$L \leq 100 \implies 3s \geq 292 - 100 = 192 \implies s \geq \frac{192}{3} = 64$$

So s ranges from 64 to 72, giving $72 - 64 + 1 = 9$ values.

Step 3: Final Answer

$$\boxed{9} \text{ possible values}$$

 Name: ____________________

Solution to Problem 4

Find the units digit of 44^{150}.

Step 1: Find the repeating pattern of units digits
Calculate the first few powers of 44 and observe the units digit:
The units digits repeat in the pattern: 4, 6

Step 2: Find the position in the cycle
The cycle length is 2. To find which digit in the pattern:

$$150 \bmod 2 = 0$$

The units digit is the 2nd element of the pattern, which is 6.

Step 3: Final Answer
The solution is

$$\boxed{6}$$

Solution to Problem 5

Step 1: Understand the Repunit
Let R_n be the number consisting of n ones. We want R_{11} mod 30.
Note the recurrence: $R_{n+1} = 10 \cdot R_n + 1$, so the remainders satisfy:
$r_{n+1} = (10 \cdot r_n + 1) \bmod 30$, with $r_1 = 1$.

Step 2: Build the Sequence of Remainders
Applying the recurrence step by step:
$r_1 = (10 \cdot 0 + 1) \bmod 30 = 1$
$r_2 = (10 \cdot 1 + 1) \bmod 30 = 11$
$r_3 = (10 \cdot 11 + 1) \bmod 30 = 21$
$r_4 = (10 \cdot 21 + 1) \bmod 30 = 1$
$r_4 = 1$ was seen before at r_1 – the sequence repeats with period 3.

Step 3: Use the Repeating Pattern
The remainders repeat with period 3 starting from r_1.
$11 - 1 = 10 = 3 \times 3 + 1$, so $r_{11} = r_2$.
Therefore $r_{11} = 11$.

Step 4: Final Answer
The remainder is

$$\boxed{11}$$

Name: ______________________________

Solution to Problem 6

Step 1: Analyze Each Component
For each entry $v \in \{-1, 0, 1\}$, the contribution v^k depends on the exponent k. With $v \in \{-1, 0, 1\}$:

$$0^k = 0, \quad 1^k = 1, \quad (-1)^k = 1 \text{ if } k \text{ even}, \quad (-1)^k = -1 \text{ if } k \text{ odd}$$

The contributions and multiplicities for each position are:

$$\begin{aligned}
a^3 &: \quad -1(\text{from 1 value}), 0(\text{from 1 value}), 1(\text{from 1 value}) \\
b^6 &: \quad 0(\text{from 1 value}), 1(\text{from 2 values}) \\
c^4 &: \quad 0(\text{from 1 value}), 1(\text{from 2 values}) \\
d^1 &: \quad -1(\text{from 1 value}), 0(\text{from 1 value}), 1(\text{from 1 value}) \\
e^5 &: \quad -1(\text{from 1 value}), 0(\text{from 1 value}), 1(\text{from 1 value}) \\
f^2 &: \quad 0(\text{from 1 value}), 1(\text{from 2 values})
\end{aligned}$$

Step 2: Bound the Sum
Summing the maximum contribution at each position gives the maximum possible value, and similarly for the minimum:

$$\min = -3, \quad \max = 6$$

Step 3: Case Analysis
We enumerate ordered choices of contribution from each position. For each position, list (contribution, multiplicity); a tuple of contributions summing to 1 contributes the product of its multiplicities.

$$\begin{gathered}
(1, 1, 1, -1, -1, 0) : 1 \cdot 2 \cdot 2 \cdot 1 \cdot 1 \cdot 1 = 4 \\
(1, 1, 0, 0, -1, 0) : 1 \cdot 2 \cdot 1 \cdot 1 \cdot 1 \cdot 1 = 2 \\
(1, 0, 1, 0, -1, 0) : 1 \cdot 1 \cdot 2 \cdot 1 \cdot 1 \cdot 1 = 2 \\
(0, 1, 1, 0, -1, 0) : 1 \cdot 2 \cdot 2 \cdot 1 \cdot 1 \cdot 1 = 4 \\
(1, 0, 0, 1, -1, 0) : 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 = 1 \\
(0, 1, 0, 1, -1, 0) : 1 \cdot 2 \cdot 1 \cdot 1 \cdot 1 \cdot 1 = 2 \\
(0, 0, 1, 1, -1, 0) : 1 \cdot 1 \cdot 2 \cdot 1 \cdot 1 \cdot 1 = 2 \\
(-1, 1, 1, 1, -1, 0) : 1 \cdot 2 \cdot 2 \cdot 1 \cdot 1 \cdot 1 = 4 \\
(1, 1, 0, -1, 0, 0) : 1 \cdot 2 \cdot 1 \cdot 1 \cdot 1 \cdot 1 = 2 \\
(1, 0, 1, -1, 0, 0) : 1 \cdot 1 \cdot 2 \cdot 1 \cdot 1 \cdot 1 = 2 \\
(0, 1, 1, -1, 0, 0) : 1 \cdot 2 \cdot 2 \cdot 1 \cdot 1 \cdot 1 = 4 \\
(1, 0, 0, 0, 0, 0) : 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 = 1 \\
(0, 1, 0, 0, 0, 0) : 1 \cdot 2 \cdot 1 \cdot 1 \cdot 1 \cdot 1 = 2
\end{gathered}$$

Name: ____________________

$$(0,0,1,0,0,0) : 1 \cdot 1 \cdot 2 \cdot 1 \cdot 1 \cdot 1 = 2$$
$$(-1,1,1,0,0,0) : 1 \cdot 2 \cdot 2 \cdot 1 \cdot 1 \cdot 1 = 4$$
$$(0,0,0,1,0,0) : 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 = 1$$
$$(-1,1,0,1,0,0) : 1 \cdot 2 \cdot 1 \cdot 1 \cdot 1 \cdot 1 = 2$$
$$(-1,0,1,1,0,0) : 1 \cdot 1 \cdot 2 \cdot 1 \cdot 1 \cdot 1 = 2$$
$$(1,0,0,-1,1,0) : 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 = 1$$
$$(0,1,0,-1,1,0) : 1 \cdot 2 \cdot 1 \cdot 1 \cdot 1 \cdot 1 = 2$$
$$(0,0,1,-1,1,0) : 1 \cdot 1 \cdot 2 \cdot 1 \cdot 1 \cdot 1 = 2$$
$$(-1,1,1,-1,1,0) : 1 \cdot 2 \cdot 2 \cdot 1 \cdot 1 \cdot 1 = 4$$
$$(0,0,0,0,1,0) : 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 = 1$$
$$(-1,1,0,0,1,0) : 1 \cdot 2 \cdot 1 \cdot 1 \cdot 1 \cdot 1 = 2$$
$$(-1,0,1,0,1,0) : 1 \cdot 1 \cdot 2 \cdot 1 \cdot 1 \cdot 1 = 2$$
$$(-1,0,0,1,1,0) : 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 = 1$$
$$(1,1,0,-1,-1,1) : 1 \cdot 2 \cdot 1 \cdot 1 \cdot 1 \cdot 2 = 4$$
$$(1,0,1,-1,-1,1) : 1 \cdot 1 \cdot 2 \cdot 1 \cdot 1 \cdot 2 = 4$$
$$(0,1,1,-1,-1,1) : 1 \cdot 2 \cdot 2 \cdot 1 \cdot 1 \cdot 2 = 8$$
$$(1,0,0,0,-1,1) : 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 2 = 2$$
$$(0,1,0,0,-1,1) : 1 \cdot 2 \cdot 1 \cdot 1 \cdot 1 \cdot 2 = 4$$
$$(0,0,1,0,-1,1) : 1 \cdot 1 \cdot 2 \cdot 1 \cdot 1 \cdot 2 = 4$$
$$(-1,1,1,0,-1,1) : 1 \cdot 2 \cdot 2 \cdot 1 \cdot 1 \cdot 2 = 8$$
$$(0,0,0,1,-1,1) : 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 2 = 2$$
$$(-1,1,0,1,-1,1) : 1 \cdot 2 \cdot 1 \cdot 1 \cdot 1 \cdot 2 = 4$$
$$(-1,0,1,1,-1,1) : 1 \cdot 1 \cdot 2 \cdot 1 \cdot 1 \cdot 2 = 4$$
$$(1,0,0,-1,0,1) : 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 2 = 2$$
$$(0,1,0,-1,0,1) : 1 \cdot 2 \cdot 1 \cdot 1 \cdot 1 \cdot 2 = 4$$
$$(0,0,1,-1,0,1) : 1 \cdot 1 \cdot 2 \cdot 1 \cdot 1 \cdot 2 = 4$$
$$(-1,1,1,-1,0,1) : 1 \cdot 2 \cdot 2 \cdot 1 \cdot 1 \cdot 2 = 8$$
$$(0,0,0,0,0,1) : 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 2 = 2$$
$$(-1,1,0,0,0,1) : 1 \cdot 2 \cdot 1 \cdot 1 \cdot 1 \cdot 2 = 4$$
$$(-1,0,1,0,0,1) : 1 \cdot 1 \cdot 2 \cdot 1 \cdot 1 \cdot 2 = 4$$
$$(-1,0,0,1,0,1) : 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 2 = 2$$
$$(0,0,0,-1,1,1) : 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 2 = 2$$
$$(-1,1,0,-1,1,1) : 1 \cdot 2 \cdot 1 \cdot 1 \cdot 1 \cdot 2 = 4$$

$$(-1,0,1,-1,1,1) : 1 \cdot 1 \cdot 2 \cdot 1 \cdot 1 \cdot 2 = 4$$
$$(-1,0,0,0,1,1) : 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 2 = 2$$
$$\text{Total} = 144$$

Step 4: Final Answer

$$\boxed{144}$$

Name: ______________________________

Solution to Problem 7

To find the number of factors of 65^2:

Step 1: Prime factorization
Express 65 in terms of its prime factors:

$$65 = 5 \cdot 13$$

Thus,

$$65^2 = 5^2 \cdot 13^2$$

Step 2: Use the factor formula
For a number $n = p_1^{e_1} \cdot p_2^{e_2} \cdots$, the number of factors is $(e_1 + 1)(e_2 + 1) \cdots$. For $n = 65^2$, the number of factors is:

$$(2 + 1) \cdot (2 + 1) = 9$$

Step 3: Final Answer
The number of factors is $\boxed{9}$.

Name: ________________________________

Solution to Problem 8

Step 1: Identify the Source of Trailing Zeros
Each trailing zero in 105! comes from a factor of $10 = 2 \times 5$.
Since factors of 2 are far more plentiful than factors of 5, the number of trailing zeros equals the exponent of 5 in $n!$.

Step 2: Apply Legendre's Formula for $p = 5$

$$\nu_5(105!) = \left\lfloor \frac{105}{5} \right\rfloor + \left\lfloor \frac{105}{25} \right\rfloor + \left\lfloor \frac{105}{125} \right\rfloor + \cdots$$

Step 3: Compute Each Term

$$\left\lfloor \frac{105}{5} \right\rfloor = 21 \text{ (multiples of 5)}$$
$$\left\lfloor \frac{105}{25} \right\rfloor = 4 \text{ (multiples of 25)}$$

Step 4: Sum the Terms

$$\nu_5(105!) = 21 + 4 = 25$$

Step 5: Final Answer

$$\boxed{25}$$

Name: ________________________

Solution to Problem 9

Step 1: Analyze Each Component
For each entry $v \in \{-1, 0, 1\}$, the contribution v^k depends on the exponent k. With $v \in \{-1, 0, 1\}$:

$$0^k = 0, \quad 1^k = 1, \quad (-1)^k = 1 \text{ if } k \text{ even}, \quad (-1)^k = -1 \text{ if } k \text{ odd}$$

The contributions and multiplicities for each position are:

$$\begin{aligned}
a^3 &: \quad -1(\text{from 1 value}), 0(\text{from 1 value}), 1(\text{from 1 value}) \\
b^5 &: \quad -1(\text{from 1 value}), 0(\text{from 1 value}), 1(\text{from 1 value}) \\
c^2 &: \quad 0(\text{from 1 value}), 1(\text{from 2 values}) \\
d^6 &: \quad 0(\text{from 1 value}), 1(\text{from 2 values}) \\
e^4 &: \quad 0(\text{from 1 value}), 1(\text{from 2 values}) \\
f^1 &: \quad -1(\text{from 1 value}), 0(\text{from 1 value}), 1(\text{from 1 value})
\end{aligned}$$

Step 2: Bound the Sum
Summing the maximum contribution at each position gives the maximum possible value, and similarly for the minimum:

$$\min = -3, \quad \max = 6$$

Step 3: Case Analysis
We enumerate ordered choices of contribution from each position. For each position, list (contribution, multiplicity); a tuple of contributions summing to 3 contributes the product of its multiplicities.

$$\begin{gathered}
(1,1,1,1,0,-1) : 1 \cdot 1 \cdot 2 \cdot 2 \cdot 1 \cdot 1 = 4 \\
(1,1,1,0,1,-1) : 1 \cdot 1 \cdot 2 \cdot 1 \cdot 2 \cdot 1 = 4 \\
(1,1,0,1,1,-1) : 1 \cdot 1 \cdot 1 \cdot 2 \cdot 2 \cdot 1 = 4 \\
(1,0,1,1,1,-1) : 1 \cdot 1 \cdot 2 \cdot 2 \cdot 2 \cdot 1 = 8 \\
(0,1,1,1,1,-1) : 1 \cdot 1 \cdot 2 \cdot 2 \cdot 2 \cdot 1 = 8 \\
(1,1,1,0,0,0) : 1 \cdot 1 \cdot 2 \cdot 1 \cdot 1 \cdot 1 = 2 \\
(1,1,0,1,0,0) : 1 \cdot 1 \cdot 1 \cdot 2 \cdot 1 \cdot 1 = 2 \\
(1,0,1,1,0,0) : 1 \cdot 1 \cdot 2 \cdot 2 \cdot 1 \cdot 1 = 4 \\
(0,1,1,1,0,0) : 1 \cdot 1 \cdot 2 \cdot 2 \cdot 1 \cdot 1 = 4 \\
(1,1,0,0,1,0) : 1 \cdot 1 \cdot 1 \cdot 1 \cdot 2 \cdot 1 = 2 \\
(1,0,1,0,1,0) : 1 \cdot 1 \cdot 2 \cdot 1 \cdot 2 \cdot 1 = 4 \\
(0,1,1,0,1,0) : 1 \cdot 1 \cdot 2 \cdot 1 \cdot 2 \cdot 1 = 4 \\
(1,0,0,1,1,0) : 1 \cdot 1 \cdot 1 \cdot 2 \cdot 2 \cdot 1 = 4
\end{gathered}$$

$$(0,1,0,1,1,0) : 1 \cdot 1 \cdot 1 \cdot 2 \cdot 2 \cdot 1 = 4$$
$$(1,-1,1,1,1,0) : 1 \cdot 1 \cdot 2 \cdot 2 \cdot 2 \cdot 1 = 8$$
$$(0,0,1,1,1,0) : 1 \cdot 1 \cdot 2 \cdot 2 \cdot 2 \cdot 1 = 8$$
$$(-1,1,1,1,1,0) : 1 \cdot 1 \cdot 2 \cdot 2 \cdot 2 \cdot 1 = 8$$
$$(1,1,0,0,0,1) : 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 = 1$$
$$(1,0,1,0,0,1) : 1 \cdot 1 \cdot 2 \cdot 1 \cdot 1 \cdot 1 = 2$$
$$(0,1,1,0,0,1) : 1 \cdot 1 \cdot 2 \cdot 1 \cdot 1 \cdot 1 = 2$$
$$(1,0,0,1,0,1) : 1 \cdot 1 \cdot 1 \cdot 2 \cdot 1 \cdot 1 = 2$$
$$(0,1,0,1,0,1) : 1 \cdot 1 \cdot 1 \cdot 2 \cdot 1 \cdot 1 = 2$$
$$(1,-1,1,1,0,1) : 1 \cdot 1 \cdot 2 \cdot 2 \cdot 1 \cdot 1 = 4$$
$$(0,0,1,1,0,1) : 1 \cdot 1 \cdot 2 \cdot 2 \cdot 1 \cdot 1 = 4$$
$$(-1,1,1,1,0,1) : 1 \cdot 1 \cdot 2 \cdot 2 \cdot 1 \cdot 1 = 4$$
$$(1,0,0,0,1,1) : 1 \cdot 1 \cdot 1 \cdot 1 \cdot 2 \cdot 1 = 2$$
$$(0,1,0,0,1,1) : 1 \cdot 1 \cdot 1 \cdot 1 \cdot 2 \cdot 1 = 2$$
$$(1,-1,1,0,1,1) : 1 \cdot 1 \cdot 2 \cdot 1 \cdot 2 \cdot 1 = 4$$
$$(0,0,1,0,1,1) : 1 \cdot 1 \cdot 2 \cdot 1 \cdot 2 \cdot 1 = 4$$
$$(-1,1,1,0,1,1) : 1 \cdot 1 \cdot 2 \cdot 1 \cdot 2 \cdot 1 = 4$$
$$(1,-1,0,1,1,1) : 1 \cdot 1 \cdot 1 \cdot 2 \cdot 2 \cdot 1 = 4$$
$$(0,0,0,1,1,1) : 1 \cdot 1 \cdot 1 \cdot 2 \cdot 2 \cdot 1 = 4$$
$$(-1,1,0,1,1,1) : 1 \cdot 1 \cdot 1 \cdot 2 \cdot 2 \cdot 1 = 4$$
$$(0,-1,1,1,1,1) : 1 \cdot 1 \cdot 2 \cdot 2 \cdot 2 \cdot 1 = 8$$
$$(-1,0,1,1,1,1) : 1 \cdot 1 \cdot 2 \cdot 2 \cdot 2 \cdot 1 = 8$$
$$\text{Total} = 147$$

Step 4: Final Answer

$$\boxed{147}$$

Solution to Problem 10

Step 1: Rewrite the Equation
Starting from the equation:

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{10}$$

Multiply both sides by $10ab$:

$$10b + 10a = ab$$

Rearrange and apply Simon's Favorite Factoring Trick by adding 100 to both sides.

$$ab - 10a - 10b = 0 \implies (a - 10)(b - 10) = 10^2 = 100$$

Step 2: List Divisor Pairs
Find all ordered factorizations $100 = d_1 \times d_2$. Since the equation is symmetric in a and b, we only count unordered pairs, so we require $d_1 \leq d_2$:
$(1, 100) \Rightarrow a = 11,\ b = 110;\quad (2, 50) \Rightarrow a = 12,\ b = 60;\quad (4, 25) \Rightarrow a = 14,\ b = 35;\quad (5, 20) \Rightarrow a = 15,\ b = 30;\quad (10, 10) \Rightarrow a = 20,\ b = 20.$

Step 3: Sum of a+b Values

$$(11 + 110) + (12 + 60) + (14 + 35) + (15 + 30) + (20 + 20) = 327$$

$$\boxed{327}$$