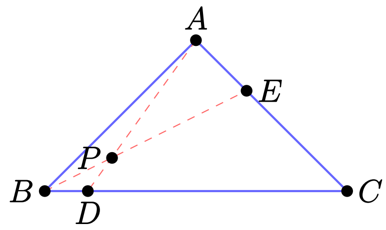
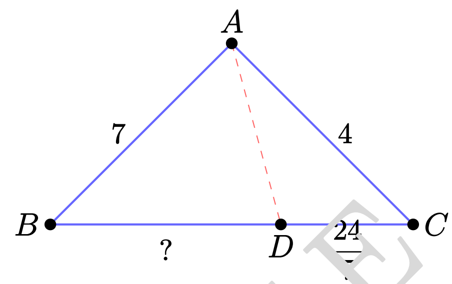
Name: ____________________

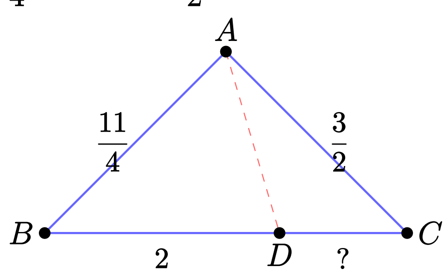
(ID:117) 1. In triangle ABC, cevian AD is drawn with D on BC such that $BD : DC = 1 : 6$. Cevian BE is drawn with E on CA such that $CE : EA = 2 : 1$. The two cevians meet at point P. Find the ratio $AP : PD$.



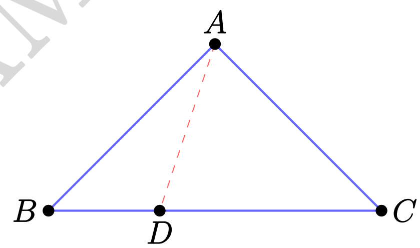
(ID:118) 2. In triangle ABC, AD is the angle bisector of $\angle BAC$. Given $AB = 7$, $AC = 4$, and $DC = \frac{24}{7}$, find BD.



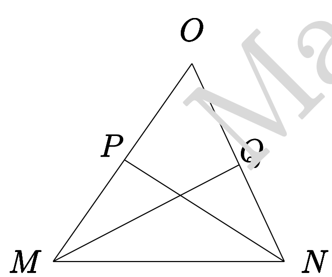
(ID:119) 3. In triangle ABC, AD is the angle bisector of $\angle BAC$. Given $AB = \frac{11}{4}$, $AC = \frac{3}{2}$, and $BD = 2$, find DC.



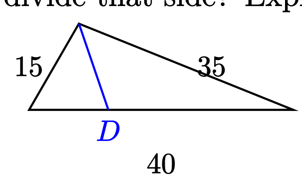
(ID:120) 4. In triangle ABC, AD bisects $\angle BAC$. If $AB = 8$, $AC = 16$, and $BC = 19$, find BD.



(ID:181) 5. In triangle MNO, $\angle M$=55°, $\angle N$=65°, NP bisects $\angle N$ and MQ bisects $\angle M$. What is the measure of $\angle MQN$?

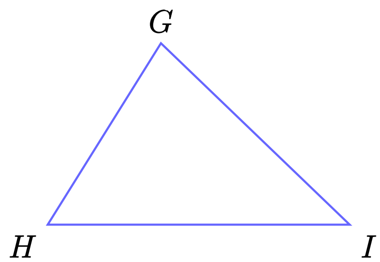


(ID:738) 6. Cameron's triangle has two sides measuring 15 and 35. The angle bisector from their shared vertex meets the opposite side (of length 40). Into what ratio does it divide that side? Express as $p : q$.

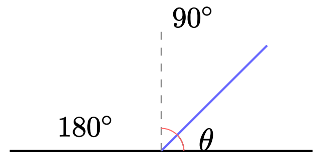


Name: ____________________

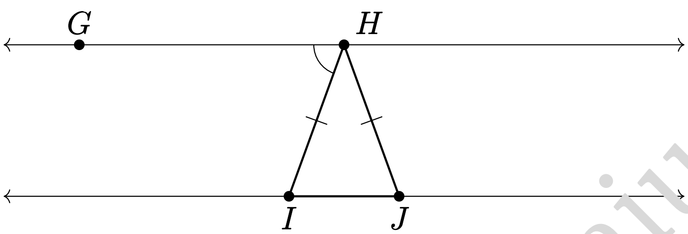
(ID:10) 7. In triangle GHI, angle G is 5 times angle H, and angle I is 5° more than angle H. What is the measure of angle I?



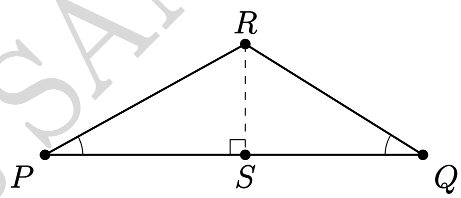
(ID:105) 8. The measure of an angle whose supplement is $2\frac{1}{9}$ times that of its complement is?



(ID:616) 9. Lines GH and IJ are parallel. Triangle HIJ is isosceles with $HI = HJ$, where H lies on line GH and IJ is on the other parallel line. If $\angle GHI = 70°$, find $\angle IHJ$.



(ID:617) 10. In triangle PQR, an altitude is drawn from R to side PQ, meeting it at point S. If $\angle P = 29°$ and $\angle Q = 32°$, find $\angle PRS$.



Math Genius
Geometry Basics

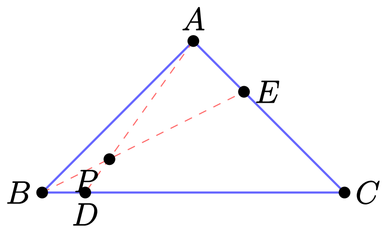
Name: ________________________

Answer Key

1. 7:2
2. 6
3. $\frac{12}{11}$
4. $\frac{19}{3}$
5. 87.5°
6. 3:7
7. 30°
8. 9
9. 40
10. 61

Name: ____________________

Solution to Problem 1



In triangle ABC, cevians AD (with D on BC) and BE (with E on CA) meet at point P.

Step 1: Assign masses from BD:DC
$BD : DC = 1 : 6$, so set $m(B) = 6$ and $m(C) = 1$ (masses inversely proportional to segment lengths).

$$m(B) = 6, \quad m(C) = 1$$

Step 2: Assign mass to A from CE:EA
$CE : EA = 2 : 1$, so $m(C)/m(A) = 1/2$, giving $m(A) = 2 \times m(C) = 2 \times 1 = 2$.

$$m(A) = 2$$

Step 3: Find AP:PD
The mass at D equals $m(B) + m(C)$ (since D is on BC). Then $AP : PD = m(D) : m(A)$.

$$m(D) = m(B) + m(C) = 6 + 1 = 7$$

$$AP : PD = \frac{m(D)}{m(A)} = \frac{7}{2}$$

Step 4: Final Answer

$$\boxed{AP : PD = 7 : 2}$$

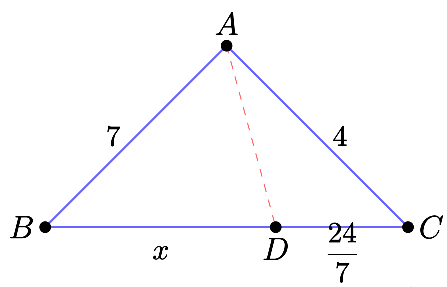
Name: ____________________

Solution to Problem 2

Step 1: State the Angle Bisector Theorem

If AD is the angle bisector of $\angle BAC$ in triangle ABC (with D on BC), then:

$$\frac{BD}{DC} = \frac{AB}{AC}$$



Step 2: Set Up the Proportion

With $AB = 7$, $AC = 4$, $DC = \frac{24}{7}$, $BD = x$:

$$\frac{x}{\frac{24}{7}} = \frac{7}{4}$$

Step 3: Cross-Multiply and Solve

$$x \cdot 4 = \frac{24}{7} \cdot 7$$
$$4x = 24$$
$$x = \frac{24}{4} = 6$$

Step 4: Final Answer

$$\boxed{6}$$

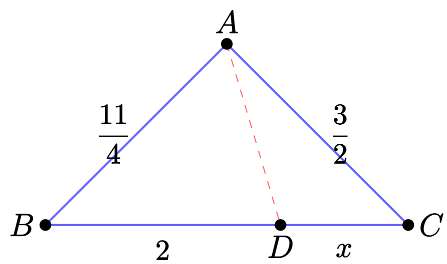
Name: ______________________

Solution to Problem 3

Step 1: State the Angle Bisector Theorem

The Angle Bisector Theorem states that if AD is the angle bisector of $\angle BAC$ in triangle ABC, then:

$$\frac{BD}{DC} = \frac{AB}{AC}$$



Step 2: Set Up the Proportion

With $AB = \frac{11}{4}$, $AC = \frac{3}{2}$, and $BD = 2$:

$$\frac{2}{x} = \frac{\frac{11}{4}}{\frac{3}{2}}$$

Step 3: Solve for the Unknown Segment

$$2 \cdot \frac{3}{2} = x \cdot \frac{11}{4}$$

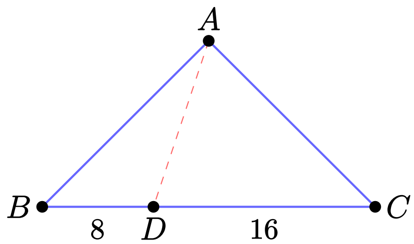
$$x = \frac{2 \cdot \frac{3}{2}}{\frac{11}{4}} = \frac{12}{11}$$

Step 4: Final Answer

$$\boxed{\frac{12}{11}}$$

Name: ______________________________

Solution to Problem 4



Step 1: State the Angle Bisector Theorem
The Angle Bisector Theorem states that the bisector of an angle in a triangle divides the opposite side in the ratio of the adjacent sides:

$$\frac{BD}{DC} = \frac{AB}{AC}$$

Step 2: Set Up the Proportion
With $AB = 8$, $AC = 16$, and $BC = 19$:

$$\frac{BD}{DC} = \frac{8}{16}$$

Step 3: Express Using the Total
Since $BD + DC = BC = 19$, write $DC = 19 - BD$:

$$\frac{BD}{19 - BD} = \frac{8}{16}$$

Step 4: Solve for BD

$$BD \cdot 16 = 8 \cdot (19 - BD)$$
$$BD \cdot 16 + BD \cdot 8 = 152$$
$$BD \cdot 24 = 152$$
$$BD = \frac{152}{24} = \frac{19}{3}$$

Step 5: Final Answer

$$\boxed{\frac{19}{3}}$$

Name: ______________________________

Solution to Problem 5

To find the measure of $\angle MQNintriangleMNO, where\angle M = 55^\circ$, $\angle N = 65^\circ$, MQ bisects $\angle M$, and NP bisects $\angle N$:

Step 1: Angle bisectors
Line MQ bisects $\angle M, so\angle NMQ = \angle OMQ = \frac{55^\circ}{2}$.
Line NP bisects $\angle N, so\angle MNP = \angle ONP = \frac{65^\circ}{2}$.

Step 2: Analyze triangle MQN
Consider triangle MQN. Its three interior angles sum to 180°:

$$\angle QMN + \angle MNQ + \angle MQN = 180^\circ$$

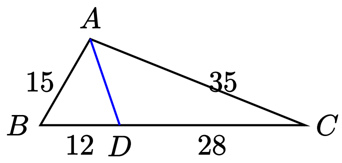
Since point Q lies on side NO, ray NQ coincides with ray NO, so $\angle MNQ = \angle MNO = 65^\circ$.
Also, $\angle QMN = \frac{55^\circ}{2}$ (since MQ bisects $\angle M$).

Step 3: Use angle relationships
Solve for the remaining angle:
$\angle MQN = 180 - \frac{55}{2} - 65 = 87.5^\circ$

Step 4: Final Answer
The measure of $\angle MQNis\boxed{87.5}$.

Name: ______________________________

Solution to Problem 6



Step 1: Apply the Angle Bisector Theorem
The angle bisector from A divides the opposite side BC in the ratio of the two adjacent sides AB and AC:

$$\frac{BD}{DC} = \frac{AB}{AC}$$
$$\frac{BD}{DC} = \frac{15}{35} = \frac{3}{7}$$

Step 2: Verify with segment lengths
Let $BD = 3k$ and $DC = 7k$. Since $BD + DC = 40$:

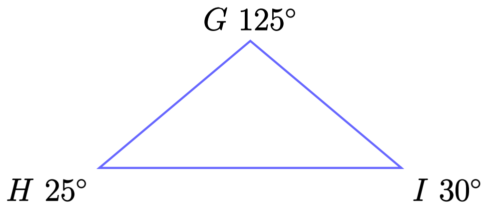
$$10k = 40 \implies k = 4$$
$$BD = 12, \quad DC = 28$$

Step 3: Final Answer
Ratio $BD : DC =$

$$\boxed{3 : 7}$$

 Name: ____________________

Solution to Problem 7



To find the measure of angle I in triangle GHI, where angle G is 5 times angle H, and angle I is 5° more than angle H:

Step 1: Set up angle relationships
Let angle H $= x°$. Then angle G $= 5x°$, and angle I $= x + 5°$.

Step 2: Sum of angles
The sum of angles in a triangle is 180°: $5x + x + (x + 5) = 180$.

Step 3: Solve for x

$$7x + 5 = 180$$
$$7x = 175$$
$$x = \frac{175}{7} = 25°.$$

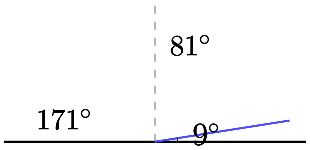
Step 4: Calculate angle I
Angle I $= 25 + 5 = 30°$.

Step 5: Final Answer

$$\boxed{30°}$$

Name: ______________________________

Solution to Problem 8



Step 1: Let the angle be x
Let the angle be $x°$.

Step 2: Write expressions for supplement and complement
Supplement: $180° - x$
Complement: $90° - x$

Step 3: Set up the equation using the given ratio

$$\frac{180 - x}{90 - x} = k$$

Step 4: Substitute the given value for k
In this problem:

$$k = \frac{19}{9}$$

Step 5: Solve for x
Cross-multiply: $180 - x = k(90 - x)$
Expand: $180 - x = 90k - kx$
Rearrange: $180 + (k - 1)x = 90k$
So: $(k - 1)x = 90k - 180$

$$x = \frac{90k - 180}{k - 1}$$

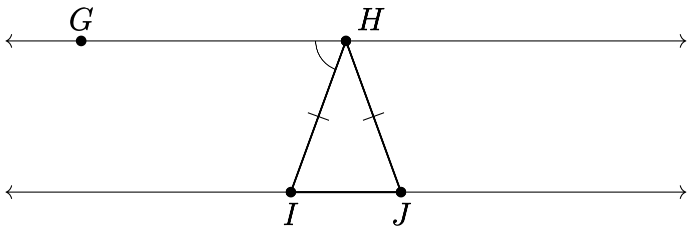
Step 6: Substitute and simplify

$$x = \frac{90 \times \frac{19}{9} - 180}{\frac{19}{9} - 1} = 9°$$

Step 7: Final Answer

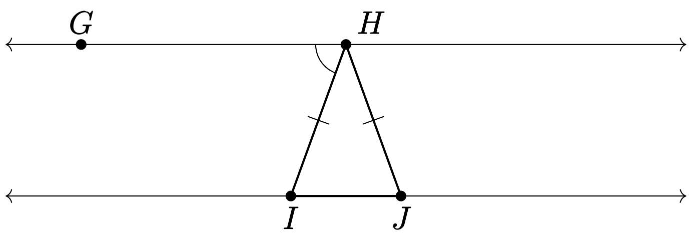
$$\boxed{9°}$$

Solution to Problem 9



Step 1: Set up the configuration
Lines GH and IJ are parallel. Vertex H of the isosceles triangle lies on the upper line, and the base IJ lies entirely on the lower line.

Step 2: Apply the Alternate Interior Angles Theorem



Line segment HI is a transversal crossing the two parallel lines.
The angle $\angle GHI = 70°$ (given) and $\angle HIJ$ are alternate interior angles, so:

$$\angle HIJ = 70°$$

Step 3: Use the isosceles triangle property
Triangle HIJ is isosceles with $HI = HJ$, so its base angles are equal:

$$\angle HIJ = \angle HJI = 70°$$

Step 4: Find the vertex angle
The angles of a triangle sum to 180°:

$$\angle IHJ + \angle HIJ + \angle HJI = 180°$$
$$\angle IHJ + 70° + 70° = 180°$$
$$\angle IHJ = 180° - 140° = 40°$$

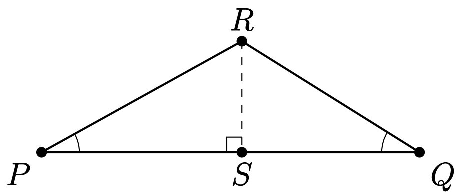
Step 5: Final Answer
The vertex angle $\angle IHJ$ measures

$$\boxed{40°}$$

Math Genius
Geometry Basics

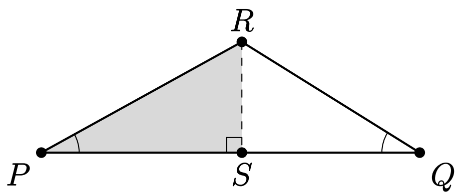
Name: ______________________________

Solution to Problem 10



Step 1: Identify the altitude
An altitude from R to PQ creates a right angle at S.
So $\angle PSR = 90°$.

Step 2: Focus on the right triangle



In right triangle PSR:
$\angle P = 29°$ (given)
$\angle PSR = 90°$ (altitude creates right angle)

Step 3: Calculate the unknown angle
The three angles in a triangle sum to 180°:

$$\angle P + \angle PSR + \angle PRS = 180°$$
$$29° + 90° + \angle PRS = 180°$$
$$\angle PRS = 61°$$

Step 4: Final Answer
Angle PRS measures

$$\boxed{61°}$$