Math Genius
Number Theory

Name: _____

(ID:424) 1. What is the sum of all prime numbers between 108 and 138 (inclusive)?

(ID:425) 2. How many twin prime pairs (primes that differ by 2) are there between 14 and 57?

(ID:426) 3. How many prime triplets of the form (p, p+2, p+6) or (p, p+4, p+6) exist between 33 and 48?

(ID:427) 4. Express 482 as the sum of two prime numbers (Goldbach's Conjecture).

(ID:428) 5. How many prime numbers are there between 76 and 122 (inclusive)?

(ID:429) 6. How many Sophie Germain primes (primes p where $2p + 1$ is also prime) are there between 64 and 79?

(ID:430) 7. How many cousin prime pairs (pairs of primes that differ by 4) are there between 68 and 135 (inclusive)?

(ID:431) 8. What is the difference in the number of prime factors (counting multiplicity) between 76 and 464?

(ID:434) 9. What is the smallest prime number greater than 472?

(ID:435) 10. What is the largest prime number less than 458?

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Answer Key

1. 617
2. 3
3. 2
4. $3 + 479$
5. 9
6. 0
7. 5
8. 2
9. 479
10. 457

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Solution to Problem 1

Find the sum of all prime numbers between 108 and 138 (inclusive).

Step 1: Identify the primes in the range
Primes in [108, 138]: 109, 113, 127, 131, 137

Step 2: Add the primes

$$109 + 113 + 127 + 131 + 137 = 617$$

Step 3: Final Answer
The sum of all primes between 108 and 138 is

$$\boxed{617}$$

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Solution to Problem 2

Count twin prime pairs (primes differing by 2) between 14 and 57.

Step 1: Understanding twin primes
Twin primes are pairs of primes that differ by exactly 2, like (3, 5) or (11, 13).
For each prime p in the range, we check whether $p + 2$ is also prime.

Step 2: List primes and identify pairs
Twin prime pairs found: (17, 19), (29, 31), (41, 43)

Step 3: Final Answer
There are

$$\boxed{3}$$

twin prime pairs

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Solution to Problem 3

Count prime triplets between 33 and 48.

Step 1: Understanding prime triplets
Prime triplets have the form $(p,\ p+2,\ p+6)$ or $(p,\ p+4,\ p+6)$.
Example: $(5, 7, 11)$ is a prime triplet since $7 = 5 + 2$ and $11 = 5 + 6$.

Step 2: Identify triplets
Triplets found: (37, 41, 43), (41, 43, 47)

Step 3: Final Answer
There are

$$\boxed{2}$$

prime triplets

 Name: _____

Solution to Problem 4

Express 482 as the sum of two prime numbers (Goldbach's Conjecture).

Step 1: Understanding Goldbach's Conjecture
Goldbach's Conjecture states that every even integer greater than 2 can be expressed as the sum of two primes.

Step 2: Find the prime pair
We need two primes p and q such that $p + q = 482$.
Test pairs of primes systematically:

Step 3: Verify the solution

$$3 + 479 = 482$$

Both 3 and 479 are prime, so this is a valid representation.

Step 4: Final Answer
$482 =$

$$\boxed{3 + 479}$$

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Solution to Problem 5

Count the prime numbers between 76 and 122 (inclusive).

Step 1: List the primes
Primes in range: 79, 83, 89, 97, 101, 103, 107, 109, 113

Step 2: Count the primes
Total count: 9 primes

Step 3: Final Answer
There are

$$\boxed{9}$$

primes

 Name: _____

Solution to Problem 6

Count Sophie Germain primes between 64 and 79.

Step 1: Understanding Sophie Germain primes
A Sophie Germain prime is a prime p where $2p + 1$ is also prime.
Example: 2 is a Sophie Germain prime because $2(2) + 1 = 5$ is prime.

Step 2: Check each prime

Step 3: Final Answer
There are

$$\boxed{0}$$

Sophie Germain primes

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Solution to Problem 7

Count cousin prime pairs (primes that differ by 4) between 68 and 135.

Step 1: Understanding cousin primes
Cousin primes are pairs of primes (p, q) where $q - p = 4$.
For example, $(3, 7)$ is a cousin prime pair since $7 - 3 = 4$ and both are prime.
Cousin primes are similar to twin primes (which differ by 2), but have a gap of 4.

Step 2: Identify cousin pairs in the range
Cousin prime pairs found: (79, 83), (97, 101), (103, 107), (109, 113), (127, 131)

Step 3: Final Answer
There are

$$\boxed{5}$$

cousin prime pairs

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Solution to Problem 8

Find the difference in the number of prime factors (counting multiplicity) between 76 and 464.

Step 1: Factor each number

$$76 = 2 \times 2 \times 19 = 2^2 \times 19 \text{ (3 prime factors)}$$
$$464 = 2 \times 2 \times 2 \times 2 \times 29 = 2^4 \times 29 \text{ (5 prime factors)}$$

Step 2: Calculate the difference

$$|3 - 5| = 2$$

Step 3: Final Answer
The difference is

$$\boxed{2}$$

 Name: _____

Solution to Problem 9

Find the smallest prime greater than 472.

Step 1: Test integers after 472
Check each integer starting from 473 for primality.

Step 2: Identify the next prime
The first prime found is 479.

Step 3: Final Answer
The next prime is

$$\boxed{479}$$

 Name: _____

Solution to Problem 10

Find the largest prime less than 458.

Step 1: Test integers before 458
Check each integer starting from 457 going downward for primality.

Step 2: Identify the previous prime
The first prime found is 457.

Step 3: Final Answer
The previous prime is

$$\boxed{457}$$